

Unparticles and holography

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We construct the holographic dual theory of unparticles. The Randall-Sundrum type hard wall model is shown to produce deconstructing particles, whose spectrum has a finite mass gap proportional to the inverse of the fifth direction segment. The introduction of new scale corresponds to setting a brane in AdS_5 . The broken conformal symmetry due to this brane is restored when it is moved to infinity. Unparticles then emerge as an infinite tower of massless particles.

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Scale invariance plays a useful role in a variety of fields in physics, from phase transitions to string theory. The bizarre feature of the scale invariant sector was studied many years ago by Banks and Zaks (\mathcal{BZ}) in the context of gauge theory [1]. They examined the infrared (IR)-stable fixed point of Yang-Mills theories with massless fermions, and found that for a proper number of fermions in a certain representation the theory is chirally invariant and has no mass gap.

But in low energy particle physics, we have been confronted by the zoo of particles with a very wide mass spectrum, which explicitly breaks the scale invariance. Thus the scale invariant sector cannot be thought as ordinary particles. Recently, Georgi proposed a possibility that such a scale invariant stuff might exist and remain hidden, with the name of "unparticles" [2]. At some high energy, there are \mathcal{BZ} fields with a nontrivial IR fixed point. Their existence and interactions with the standard model (SM) particles are nicely described by an effective theory. The effective theory involves the unparticle operators on which the \mathcal{BZ} operators are matched. For an unparticle operator of scaling dimension d_U , the unparticle appears as a non-integral number d_U of invisible massless particles. After the Georgi's suggestion, there have been a lot of phenomenological studies on unparticles [3].

Another triumph involving the scale invariance during the decade is the AdS/CFT correspondence [4, 5, 6]. According to this, a strongly coupled conformal field theory on 4D is equivalent to a gravity theory on AdS in 5D. Recent works include the construction of 5D theory equivalent to the 4D QCD, which have been very successful. The essence of the equivalence is that for every CFT operator there exists a corresponding field in AdS. We assume that this correspondence still holds for the scale invariant \mathcal{BZ} sector [7]. Since the most fundamental feature of the unparticle is the scale invariance, it is quite natural to try to apply the AdS/CFT correspondence and to figure out the holographic dual description of the unparticle in AdS. Once we accept the correspondence, then for an unparticle operator there must be a corresponding bulk field in 5D AdS. In this respect, the bottom-up approach for the AdS/QCD is a good bench-

mark.

Another reason for considering the AdS/QCD is that the unparticle operators are matched from the \mathcal{BZ} operators which originate from the massless fermions of the non-Abelian gauge theory. The theory is basically QCD, except the asymptotic freedom. Thus we can inherit many developments of the holographic QCD. What makes the theory different from QCD is the scale invariance.

As an example, consider a QCD operator $\bar{q}q$. According to the AdS/CFT correspondence, there is a bulk field ϕ in the holographic dual theory in AdS_5 . Its 5-dimensional mass is given by, in general $m_5 = (\Delta - p)(\Delta + p - 4)$, where Δ is the canonical dimension of the 4D p -form operator. And it is quite well known that the bulk field ϕ scales $\phi(x, z) \sim c_1 z^{4-\Delta} + c_2 z^\Delta$ for $z \rightarrow 0$ where z is the fifth dimension [8]. For $\bar{q}q$, $m_5 = -3$ and $\phi \sim c_1 z + c_2 z^3$. Similarly for an unparticle (scalar) operator of a non-integer dimension d_U the 5D mass is $m_5 = d_U(d_U - 4)$, and the 5D field scales with non-integral power of z .

In addition, the chiral symmetry of the \mathcal{BZ} sector in 4D is much like that of the massless QCD. In the holographic dual theory, there is a corresponding local gauge symmetry from which the 5D vector gauge bosons are naturally defined. The 4D object, dual to the 5D vector gauge boson, can be identified as the \mathcal{BZ} vector operator.

The holographic dual picture of AdS/QCD can be well explained by a simple Randall-Sundrum (RS) [9] type setup [10, 11, 12]. Especially in the RS1-type background, there are two branes at $z = \epsilon$ and $z = z_c$. The quantity $1/z$ plays a role of the renormalization scale in 4D. The brane located at $z = \epsilon (\rightarrow 0)$ (UV brane) puts a UV cutoff ($\sim 1/\epsilon$) in the 4D theory, while the one at $z = z_c$ (IR brane) determines the typical mass scale of QCD, $\Lambda_{\text{QCD}} \sim 1/z_c$.

A very similar correspondence can also be applied to the unparticles. In the framework of [2], there are two relevant scales in the unparticle physics. One is a very high energy scale M_U . The SM fields and the \mathcal{BZ} fields interact via the exchange of heavy particles with the large mass scale M_U . Thus the scale M_U is the UV cutoff where a new physics appears. Below the scale M_U , the inter-

actions between the SM and \mathcal{BZ} fields are described by the nonrenormalizable couplings suppressed by powers of M_U . Then through the dimensional transmutation the renormalizable couplings of the \mathcal{BZ} fields induce another scale Λ_U where the scale-invariant \mathcal{BZ} sector appears. Below the scale Λ_U we construct an effective theory in the context of unparticle operators onto which the \mathcal{BZ} operators match at Λ_U . Since the theory enters the conformal regime at Λ_U , we build a brane in 5D dual theory located at $z = z_0 \sim 1/\Lambda_U$. In the region $z > z_0$, one can expect the AdS/CFT correspondence works well because the 4D theory is really conformal with the unparticle operators. We will call this a " \mathcal{U} brane".

Recently an interesting picture for unparticles is suggested [7]. Here the unparticles are "deconstructed" to be an infinite tower of particles of different masses. There is a mass gap of order Δ_m which will be sent to zero. The true unparticles appear when Δ_m vanishes. Because the nonvanishing parameter Δ_m introduces a scale in the theory, one may think of Δ_m as a scale-invariance breaking, or non-conformal scale Λ_U . In a 5D description, this non-conformal scale corresponds to an IR brane located at $z = z_m \sim 1/\Delta_m \sim 1/\Lambda_U$. We call this a " \mathcal{U} brane". From the study of AdS/QCD, it is well known that this kind of hard-wall model produces the mass spectrum $m_n^2 \sim n^2/z_m^2$. Now we expect that a similar mass spectrum occurs in the $z_0 \sim 1/\Lambda_U < z < z_m \sim 1/\Lambda_U$ model. The existence of the \mathcal{U} brane at $z = z_m$ represents a departure from AdS. The corresponding effect on 4D theory is the breaking of a scale invariance for $z > z_0$, which means a nonzero mass gap ($\sim 1/z_m \sim \Lambda_U$) in the spectrum. But the scale invariance is restored when $z_m \rightarrow \infty$ with the vanishing mass gap $\Lambda_U \rightarrow 0$, and the infinite tower of massless particles are identified as unparticles.

We start with the simple 5D AdS metric of RS type,

$$ds^2 = \frac{1}{z^2} (dx^\mu dx_\mu - dz^2) , \quad (1)$$

with the \mathcal{U} and \mathcal{U} branes at $z = z_0$ and $z = z_m$ respectively. In the 5D bulk there are left(right)-handed gauge bosons $A_{L(R)} = A_{L(R)}^a t^a$ of the gauge symmetry $SU(3)_L \otimes SU(3)_R$, and a scalar field $\Phi = S \exp(i2\pi^a t^a)$, where t^a are the $SU(N)$ generators. Here S is a real scalar while π^a are real pseudoscalars. The 5D action of these fields is given by $S_5 = \int d^4x \int dz \mathcal{L}_5$ with

$$\mathcal{L}_5 = \sqrt{g} \text{Tr} \left\{ -\frac{1}{4g_5^2} (F_L^2 + F_R^2) + |D\Phi|^2 - M_\Phi^2 |\Phi|^2 \right\} , \quad (2)$$

where $g = \det(g_{MN})$, $D_M = \partial_M \Phi - iA_{LM} \Phi + i\Phi A_{RM}$ ($M = \mu, z$) and $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$. Here g_5 is the 5D gauge coupling which can be matched onto the 4D theory to be $g_5^2 = 12\pi^2/N_c$ where N_c is the number of color [11].

Note that the 5D mass of the vector gauge boson in the holographic QCD vanishes for $\Delta = 3$ and $p = 1$. It means

that the local gauge symmetry in 5D is preserved. But for a vector boson corresponding to the vector unparticle operator with dimension d_U , its 5D mass will not vanish in general: $M_V^2 = (d_U - 1)(d_U - 3)$. We assume that $1 < d_U < 2$ [2]. Thus in the 5D holographic model of unparticles the gauge symmetry is broken, though we do not know the details about the breaking mechanism. We now construct the 5D Lagrangian with the gauge boson mass term:

$$\mathcal{L}_5^U = \sqrt{g} \text{Tr} \left\{ -\frac{1}{4g_5^2} (2F_V^2 - 4M_V^2 V^2) + |D\Phi|^2 - M_\Phi^2 |\Phi|^2 \right\} , \quad (3)$$

where $V_M = (A_L + A_R)/2$. We choose $V_z(x, z) = 0$ gauge. Here the possible axial vector part is dropped. Fluctuations around the vacuum expectation value of S give the real scalar degree of freedom σ : $S = v(z)/2 + \sigma$. The equation of motion for σ is

$$\left(\partial_z^2 - \frac{3}{z} \partial_z + m_S^2 - \frac{M_\Phi^2}{z^2} \right) \sigma = 0 . \quad (4)$$

The general solution is

$$\sigma(z) = c_1 z^2 J_{d_U-2}(m_S z) + c_2 z^2 Y_{d_U-2}(m_S z) . \quad (5)$$

At $z = z_0$, we impose $\sigma(z_0) = 0$ as a boundary condition [11, 12]. The wavefunction becomes

$$\begin{aligned} \sigma(z) &= c_1 z^2 \left[J_{d_U-2}(m_S z) - \frac{J_{d_U-2}(m_S z_0)}{Y_{d_U-2}(m_S z_0)} Y_{d_U-2}(m_S z) \right] \\ &\rightarrow c_1 z^2 J_{d_U-2}(m_S z) \quad \text{as } z_0 \rightarrow 0 . \end{aligned} \quad (6)$$

From the boundary condition at $z = z_m$, the mass spectrum of m_S is obtained. We require $\sigma'(z = z_m) = 0$. For large $m_S z_m \gg 1$, the boundary condition leads to $J_{d_U-1}(m_S z_m) \simeq 0$, i.e.,

$$m_{S,n} \simeq \frac{\pi}{z_m} \left(\frac{2d-3}{4} + n \right) . \quad (7)$$

Similarly, the equation of motion for the vector boson is

$$\left(\partial_z^2 - \frac{1}{z} \partial_z + m_V^2 - \frac{M_V^2}{z^2} \right) V_M = 0 . \quad (8)$$

The solution is

$$V_M(z) = c_3 z J_{d_U-2}(m_V z) + c_4 z Y_{d_U-2}(m_V z) . \quad (9)$$

Imposing the same boundary condition, $V_M(z_0) = V'_M(z_m) = 0$, gives the same mass spectrum of m_V as m_S , though the equation of motion is slightly different:

$$m_{V,n} \simeq \frac{\pi}{z_m} \left(\frac{2d-3}{4} + n \right) . \quad (10)$$

The mass spectra (7) and (10) are what we have expected before. The behavior of $m_n \sim n/z_m$ is the same as that

of [7]. For a finite value of z_m , the spectra of $m_{S,n}$ and $m_{V,n}$ are discrete and the mass gap is finite. The scale $\sim 1/z_m$ explicitly breaks the conformal symmetry. Note that if $d_{\mathcal{U}} = 3$, $m_{V,n}$ reproduces the well known results for the vector meson spectrum [12]. The unparticles emerge in the limiting situation where $z_m \rightarrow \infty$; the scale invariance is restored and there are only infinite tower of massless particles. In this sense, the RS2-type setup is a very good holographic dual theory of unparticles in AdS_5 .

In conclusion, we have constructed the holographic dual theory of unparticles. The emergence of the scale invariance of the \mathcal{BZ} fields at $\Lambda_{\mathcal{U}}$ corresponds to the \mathcal{U} brane in the RS background. For scalar and vector unparticle operators, there are bulk fields Φ and V_M much like those in the holographic QCD. The introduction of a conformal symmetry breaking scale $\Lambda_{\mathcal{U}}$ is equivalent to putting a \mathcal{U} brane in 5D at $z_m = 1/\Lambda_{\mathcal{U}}$. This is also a scale of mass gap of the 4D spectra for Φ and V_M . The scale invariance is recovered when $z_m \rightarrow \infty$ and the unparticles are identified as the infinite tower of massless particles. The limiting case of putting \mathcal{U} brane at infinity is equivalent to the RS2 scenario, which is a good holographic description of the unparticles.

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- [1] T. Banks and A. Zaks, Nucl. Phys. B **196**, 189 (1982).
 - [2] H. Georgi, Phys. Rev. Lett. **98**, 221601 (2007) [arXiv:hep-ph/0703260]; Phys. Lett. B **650**, 275 (2007) [arXiv:0704.2457 [hep-ph]].
 - [3] K. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. Lett. **99**, 051803 (2007) [arXiv:0704.2588 [hep-ph]], Phys. Rev. D **76**, 055003 (2007) [arXiv:0706.3155 [hep-ph]]; M. Luo and G. Zhu, arXiv:0704.3532 [hep-ph]; P. J. Fox, A. Rajaraman and Y. Shirman, Phys. Rev. D **76**, 075004 (2007) [arXiv:0705.3092 [hep-ph]]; T. Kikuchi and N. Okada, arXiv:0707.0893 [hep-ph]; T. i. Hur, P. Ko and X. H. Wu, arXiv:0709.0629 [hep-ph].
 - [4] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].
 - [5] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109].
 - [6] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].
 - [7] M. A. Stephanov, Phys. Rev. D **76**, 035008 (2007) [arXiv:0705.3049 [hep-ph]].
 - [8] I. R. Klebanov and E. Witten, Nucl. Phys. B **556**, 89 (1999) [arXiv:hep-th/9905104].
 - [9] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].
 - [10] N. Arkani-Hamed, M. Porrati and L. Randall, JHEP **0108**, 017 (2001) [arXiv:hep-th/0012148].
 - [11] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005) [arXiv:hep-ph/0501128].
 - [12] L. Da Rold and A. Pomarol, Nucl. Phys. B **721**, 79 (2005) [arXiv:hep-ph/0501218]; JHEP **0601**, 157 (2006) [arXiv:hep-ph/0510268].

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